

NOVEMBER/DECEMBER 2023

GMA31/DMA31 — COMPLEX ANALYSIS – I

Time : Three hours

Maximum : 75 marks



SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define analytic.
2. What do you mean by simply connected?
3. Define reparameterization.
4. State weak form of Cauchy's theorem.
5. Write invariance property.
6. State circle-preserving property.
7. State Liouville's theorem.
8. Prove that every automorphism $f : \Delta \rightarrow \Delta$ with $f(0) = 0$ is given by $f(z) = e^{i\theta}z$.

9. Define singularity.
10. Prove that a function $f(z)$ is rational iff it is meromorphic in the extended complex plane C_∞ .

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL questions.

11. (a) Prove that a real valued function of a complex variable either has derivative zero or the derivative does not exist.

Or

- (b) State and prove antiderivative theorem.



12. (a) State and prove Gauss's mean value theorem.

Or

- (b) State and prove Cauchy – Goursat theorem.

13. (a) Write down the basic properties of Mobius Maps.

Or

- (b) Prove that the inverse of Mobius transformation is also a Mobius transformation.

14. (a) State and prove Phragmen-Lindelof theorem.

Or

- (b) Are there non-constant functions of a real variable x which are both bounded and differentiable on R ?

15. (a) Discuss about removable singularity.

Or

- (b) Prove that the range of a non-constant entire function is a dense subset of C .

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. State and prove root test.
17. State and prove Cauchy's integral theorem.
18. Prove that the cross-ratio is invariant under Mobius transformation.
19. State and prove Schwarz's lemma.
20. State and prove Riemann's removable Singularity theorem.